PORTFOLIO MANAGEMENT

IMPORTANT QUESTIONS

CLASS WORK QUESTIONS

Question 5:

We have $E_p = W_1E_1 + W_3E_3 + \ldots\ldots\ldots\ldots W_nE_n$

and for standard deviation $\sigma^2_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j$

$$\sigma^2_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j$$

Two asset portfolio

$$\sigma^2_p = w^2_1\sigma^2_1 + w^2_2\sigma^2_2 + 2\, w_1w_2\sigma_1\sigma_2\rho_{12}$$

Or

$$\sigma_p = \sqrt{w_1\sigma_1 + w_2\sigma_2 + 2\, w_1w_2\sigma_1\sigma_2\rho_{12}}$$

Substituting the respective values we get,

i. **50% of funds in each of X and Y**

$Ep = 0.50 \times 19\% + 0.50 \times 23\% = 21\%$
$\sigma^2_p = (0.50)^2(14\%)^2 + (0.50)^2(18\%)^2 + 2(0.50)(0.50)(0.16)(14\%)(18\%)$
$\sigma^2_p = 49 + 81 + 20.16 = 150.16$
$\sigma_p = 12.25\%$

ii. **75% in X and 25% in Y**

$Ep = 0.75 \times 19\% + 0.25 \times 23\% = 20\%$
$\sigma^2_P = (0.75)^2(14\%)^2 + (0.25)^2(18\%)^2 + 2(0.75)(0.25)(0.16)(14\%)(18\%)$
$\sigma^2_P = 110.25 + 20.25 + 15.12 = 145.62$
$\sigma_p = 12.07\%$

iii. **25% in X and 75% in Y**

$Ep = 0.25 \times 19\% + 0.75 \times 23\% = 22\%$
$\sigma^2_p = (0.25)^2(14\%)^2 + (0.75)^2(18\%)^2 + 2(0.25)(0.75)(0.16)(14\%)(18\%)$
$\sigma^2_p = 12.25 + 182.25 + 15.12 = 209.62$
$\sigma_p = 14.48\%$

iv. **60% in X and 40% in Y**

$Ep = 0.60 \times 19\% + 0.40 \times 23\% = 20.60\%$
$\sigma^2_P = (0.60)^2(14\%)^2 + (0.40)^2(18\%)^2 + 2(0.60)(0.40)(0.16)(14\%)(18\%)$

σ^2_P = = 70.56 + 51.84 + 19.35 = 141.75

σ_p= 11.91%

v. 35% in X and 65% in Y

Ep = 0.35 X 19% + 0.65 X 23% = 21.60%

$\sigma^2_P = (0.35)^2(14\%)^2 + (0.65)^2(18\%)^2 + 2(0.35)(0.65)(0.16)(14\%)(18\%)$

σ^2_P = 24.01 + 136.89 + 18.35 = 179.25

σ_p= 13.39%

Portfolio	(i)	(ii)	(iii)	(iv)	(v)
Return	21.00	20.00	22.00	20.60	21.60
σ	12.25	12.07	14.48	11.91	13.39

In the terms of return, we see that portfolio (iii) is the best portfolio.

In terms of risk we see that portfolio (iv) is the best portfolio.

Question 13:

i.

Probability	ABC (%)	XYZ (%)	1X2 (%)	1X3 (%)
(1)	**(2)**	**(3)**	**(4)**	**(5)**
0.20	12	16	2.40	3.2
0.25	14	10	3.50	2.5
0.25	-7	28	-1.75	7.0
0.30	28	-2	8.40	-0.6
Average return			**12.55**	**12.1**

Hence the expected return from ABC = 12.55% and XYZ is 12.1%

Probability	$(ABC-\overline{ABC})$	$(ABC-\overline{ABC})^2$	1X3	$(XYZ-\overline{XYZ})$	$(XYZ-\overline{XYZ})^2$	(1)X(6)
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	
0.20	-0.55	0.3025	0.06	3.9	15.21	3.04
0.25	1.45	2.1025	0.53	-2.1	4.41	1.10
0.25	-19.55	382.2025	95.55	15.9	252.81	63.20
0.30	15.45	238.7025	71.61	-14.1	198.81	59.64
			167.75			**126.98**

$\sigma^2_{ABC} = 167.75(\%)^2; \sigma_{ABC} = 12.95\%$

$\sigma^2_{XYZ} = 126.98(\%)^2; \sigma_{XYZ} = 11.27\%$

ii. In order to find risk of portfolio of two shares, the covariance between the two is necessary here.

Probability	$(ABC-\overline{ABC})$	$(XYZ-\overline{XYZ})$	2X3	1X4

(1)	(2)	(3)	(4)	(5)
0.20	-0.55	3.9	-2.145	-0.429
0.25	1.45	-2.1	-3.045	-0.761
0.25	-19.55	15.9	-310.845	-77.71
0.30	15.45	-14.1	-217.845	-65.35
				-144.25

$$\sigma^2_p = \left(0.5^2 \times 167.75\right) + \left(0.5^2 \times 126.98\right) + 2 \times \left(-144.25\right) \times 0.5 \times 0.5$$

$$\sigma^2_p = 41.9375 + 31.745 - 72.125$$

$$\sigma^2_p = 1.5575 \text{ or } 1.56(\%)$$

$$\sigma_p = \sqrt{1.56} = 1.25\%$$

$E(R_p) = (0.5 \times 12.55) + (0.5 \times 12.1) = 12.325\%$

Hence, the return is 12.325% with the risk of 1.25% for the portfolio. Thus the portfolio results in the reduction of risk by the combination of two shares.

iii. For constructing the minimum risk portfolio the condition to be satisfied is

$$X_{ABC} = \frac{\sigma_X^2 - r_{AX}\sigma_A\sigma_X}{\sigma_A^2 + \sigma_X^2 - 2r_{AX}\sigma_A\sigma_X} \text{ or } = \frac{\sigma_X^2 - Cov.AX}{\sigma_A^2 + \sigma_X^2 - 2Cov._{AX}}$$

σ_X = Std. Deviation of XYZ

σ_A = Std. Deviation of ABC

r_{AX}= Coefficient of Correlation between XYZ and ABC

$Cov._{AX}$ = Covariance between XYZ and ABC.

Therefore,

$$\%\ ABC = \frac{126.98 - (-144.25)}{126.98 + 167.75 - \left[2 \times (-144.25)\right]} = \frac{271.23}{583.23} = 0.46 \text{ or } 46\%$$

% ABC = 46%, XYZ = 54%

(1 – 0.46) =0.54

Question 14:

The parameters are E(R) = 15, σ = 60, and the correlation between any pair of stocks is ρ = .5.

a. The portfolio expected return is invariant to the size of the portfolio because all stocks have identical expected returns. The standard deviation of a portfolio with n = 25 stock is

$$\sigma p = \left[\sigma^2/n + \rho \times \sigma^2(n-1)/n\right]^{1/2}$$

$$= \left[60^2/25 + 0.5 \times 60^2 \times 24/25\right]^{1/2} = 43.27$$

b. Because the stocks are identical, efficient portfolios are equally weighted. To obtain a standard deviation of 43%, we need to solve for n:

$$43^2 = \frac{60^2}{n} + 0.5 \times \frac{60^2(n-1)}{n}$$

1,849n = 3,600 + 1,800n - 1,800

n = 1, 800/49 = 36.73

Thus we need 37 stock and will come in with volatility slightly under the target.

c. As n gets very large, the variance of an efficient (equally weighted) portfolio diminishes, leaving only the variance that comes from the covariances among stocks, that is

$$\sigma\rho = \sqrt{\rho \times \sigma^2} = \sqrt{0.5 \times 60^2} = 42.43$$

d. If the risk-free is 10%, then the risk premium on any size portfolio is 15% - 10% = 5%. The standard deviation of a well-diversified portfolio is (practically) 42.43%; hence the slope of the Capital Allocation Line (CAL) is

S = 5/42.43 = 0.1178

Question 15:

i. An efficient portfolio shall consist of the market portfolio and risk free securities. Accordingly, let x be the proportion of total funds invested in market portfolio then

$E(R_p) = x\ E(R_m) + (1 - x)\ R_f$

7.5% = x X 8% + (1 – x) X 5%

7.5% = 8x + 5 – 5x

2.5 = 3x

x = 5/6 i.e. 83.33%

Thus, $88\frac{1}{3}\%$ total funds should be invested in market portfolio and balance 16.67% in Risk Free Securities.

ii. Risk of above Portfolio

$$7.5 = 5.00 + \frac{8\% - 5\%}{6\%}\sigma_P$$

$$2.5 = \frac{3\%}{6\%}\sigma_P$$

$15\% = 3\%\ \sigma_P$

$\sigma_P = 5\%$

iii. Let y be the proportion of investment in market portfolio then investment in risk free securities (1 – y), then

10% = y X 8% + (1 - y) 5%

10% = 8y + 5% - 5y

5% = 3y

y = 1.66

Thus, borrow $66\frac{2}{3}\%$ of owned fund i.e. 66.67% of owned funds at risk free rate of interest of 5%.

Rs. 100000 X 66.67 % = Rs. 66,670

Now invest total fund of Rs. 1,66,670 in the market portfolio.

Risk of Portfolio

$$10\% = 5\% + \frac{8\% - 5\%}{6\%}\sigma P$$

$$5\% = \frac{3\%}{6\%}\sigma P$$

σP = 10%

Question 16:

i.

Period	R_X	R_M	$R_X - \bar{R}_X$	$R_M - \bar{R}_M$	$(R_X - \bar{R}_X)(R_M - \bar{R}_M)$	$(R_M - \bar{R}_M)^2$
1	20	22	5	10	50	100
2	22	20	7	8	56	64
3	25	18	10	6	60	36
4	21	16	6	4	24	16
5	18	20	3	8	24	64
6	-5	8	-20	-4	80	16
7	17	-6	2	-18	-36	324
8	19	5	4	-7	-28	49
9	-7	6	-22	-6	132	36
10	20	11	5	-1	-5	1
	150	**120**			**357**	**706**
	$\sum R_X$	$\sum R_M$			$\sum (R_X - \bar{R}_X)(R_M - \bar{R}_M)$	$\sum (R_M - \bar{R}_M)^2$

$\bar{R}_X = 15$ $\bar{R}_M = 12$

$$\sigma^2_M = \frac{\sum (R_M - \bar{R}_M)^2}{n} = \frac{706}{10} = 70.60$$

$$Cov_{XM} = \frac{\sum (R_X - \bar{R}_X)(R_M - \bar{R}_M)}{n} = \frac{357}{10} = 35.70$$

$$Beta_x = \frac{Cov_{XM}}{\sigma^2_M} = \frac{35.70}{70.60} = 0.505$$

Alternative Solution

Period	X	Y	Y^2	XY
1	20	22	484	440
2	22	20	400	440
3	25	18	324	450
4	21	16	256	336
5	18	20	400	360
6	-5	8	64	-40
7	17	-6	36	-102
8	19	5	25	95
9	-7	6	36	-42
10	20	11	121	220
	150	120	2146	2157
	$\overline{X}=15$	$\overline{Y}=12$		

$$=\frac{\sum XY - n\overline{XY}}{\sum X^2 - n\left(\overline{X}\right)^2}$$

$$=\frac{2157-10\times15\times12}{2146-10\times12\times12}=\frac{357}{706}=0.506$$

ii. $\overline{R}_X = 15\ \overline{R}_M = 12$

$y = \alpha + \beta X$

15 = α + 0.505 × 12

Alpha (α) = 15 – (0.505 × 12) = 8.94%

Characteristic line for security X = α + β × R_M

Where, R_M = Expected return on Market Index

∴Characteristic line for security X = 8.94 + 0.505 R_M

Question 17:

i. Calculation of Expected Return, Variance and Standard Deviation for R Ltd.

Prob. (P)	R	P x R	$(R - \overline{R})$	$(R - \overline{R})^2$	$(R - \overline{R})^2 P$
0.35	30	10.50	2	4	1.40
0.30	25	7.50	- 3	9	2.70
0.15	40	6.00	12	144	21.60
0.20	20	4.00	- 8	64	12.80
		28.00			38.50

σ = √38.50 = 6.20

ii. Calculation of Expected Return, Variance and Standard Deviation for Market

Prob. (P)	M	P x M	$(M - \bar{M})$	$(M - \bar{M})^2$	$P(M - \bar{M})^2$	$(R - \bar{R})(M - \bar{M})$	$(R - \bar{R})(M - \bar{M})P$
0.35	25	8.75	3.75	14.063	4.922	7.50	2.625
0.30	20	6.00	-1.25	1.563	0.469	3.75	1.125
0.15	30	4.50	8.75	76.563	11.484	105.00	15.75
0.20	10	2.00	- 11.25	126.563	25.313	90.00	18.00
		21.25			42.188		37.50

$\sigma = \sqrt{42.188} = 6.495$

iii. Beta Co-efficient for R Ltd. Shares

$$\beta = \frac{Cov(R,M)}{\sigma_M^2} = \frac{37.50}{42.188} = 0.888$$

Question 18:

i. The Betas of two stocks:

Aggressive stock - 40% - 4%/25% - 7% = 2
Defensive stock - 18% - 9%/25% - 7% = 0.50

Alternatively, it can also be solved by using the Characteristic Line Relationship as follows:

$R_s = \alpha + \beta R_m$

Where

α = Alpha
β = Beta
R_m= Market Return

For Aggressive Stock

$4\% = \alpha + \beta(7\%)$
$40\% = \alpha + \beta(25\%)$
$36\% = \beta(18\%)$
$\beta = 2$

For Defensive Stock

$9\% = \alpha + \beta(7\%)$
$18\% = \alpha + \beta(25\%)$
$9\% = \beta(18\%)$
$\beta = 0.50$

ii. Expected returns of the two stocks:

Aggressive stock - 0.5 x 4% + 0.5 x 40% = 22%
Defensive stock - 0.5 x 9% + 0.5 x 18% = 13.5%

iii. Expected return of market portfolio = 0.5 x 7% + 0.5% x 25% = 16%

∴ Market risk prem. = 16% - 7.5% = 8.5%
∴ SML is, required return = 7.5% + βi 8.5%

iv. $R_s = \alpha + \beta R_m$

For Aggressive Stock

22% = α_A + 2(16%)
α_A = -10%

For Defensive Stock

13.5% = α_D + 0.50(16%)
α_D = 5.5%

Note: Food for thought

Logically the alpha as per characteristic line is not useful. It would be more prudent to calculate alpha as E(R) – Re

Re of aggressive stock = 7.5 + 2×8.5 = 24.5%

E(R) of aggressive stock = 22%

Alpha = -2.5% so overpriced

Re of defensive stock = 7.5 + 0.5×8.5 = 11.75%

E(R) of defensive stock = 13.5%

Alpha = 1.75% so underpriced

Question 29:

Step 1: Sell 10m $ spot @ ₹ 48/$, getting ₹ 480m

Step 2: Invest ₹ 480m in India at a return of
6 + (9 - 6) × 1.4 = 10.2%

Step 3 : Expected Rupee inflow after 1 year = 480 × 1.102 = ₹528.96m

Step 4 : Expected Spot for buying $ = ₹ 48 × 1.04/$ = ₹ 49.92/$

Step 5 : Expected $ inflow after 1 year =549.12/49.92 = $10.596m

Step 6 : Expected Return =5.96%

Note: as per logic expected spot should have been 48/0.96 = 50

So expected return will come to 5.79%, but follow ICAI style of solution

Risk(σ_P) $= \sqrt{\text{Var (Stock)} + \text{Var(Currency)} + 2 \times r \times \text{SD stock} \times \text{SD currency}}$

$= \sqrt{(10)^2 + (8)^2 + 2 \times 0.15 \times 8 \times 10} = 13.71\%$

Question 32:

i. Equity Beta

To calculate Equity Beta first we shall calculate Weighted Average of Asset Beta as follows:

= 1.55 x 0.64 + 1.40 x 0.36
= 0.992 + 0.504 = 1.496

Now we shall compute Equity Beta using the following formula:

$$\beta_{Asset} = \beta_{Equity}\left[\frac{E}{E + D(1-t)}\right] + \beta_{Debt}\left[\frac{D(1-t)}{E + D(1-t)}\right]$$

Accordingly,

$$1.496 = \beta_{Equity}\left[\frac{500}{500 + 290}\right] + \beta_{Debt}\left[\frac{290}{500+290}\right]$$

$$1.496 = \beta_{Equity}\left[\frac{500}{790}\right] + 0.28\left[\frac{290}{790}\right]$$

$\beta_{Equity} = 2.20$

ii. Equity Beta on change in Capital Structure

Amount of Debt to be raised:

Particulars	Value (in ₹ Crore)
Total Value of Firm (Equity ₹ 500 crore + Debt ₹ 290 crore)	790
Desired Debt Equity Ratio	1.50 : 1.00
Desired Debt Level = $\frac{\text{Total Value x Debt Ratio}}{\text{Debt Ratio} + \text{Equity Ratio}}$	474
Less: Value of Existing Debt	(290)
Value of Debt to be Raised	184

Equity after Repurchase = Total value of Firm – Desired Debt Value

= ₹ 790 Crore – ₹ 474 Crore = ₹ 316 Crore

Weighted Average Beta of ABCL:

Source of Finance	Investment (in ₹ Crore)	Weight	Beta of the Division	Weighted Beta
Equity	316	0.4	$\beta_{(E = X)}$	0.4x
Debt – 1	290	0.367	0.45	0.165
Debt – 2	184	0.233	0.50	0.117
	790	**Weighted Average Beta**		**0.282 + (0.4x)**

β_{ABCL} = 0.282 + 0.4x

1.496 = 0.282 + 0.4x

0.4x = 1.496 – 0.282

X = 1.214/0.4 = 3.035

$\beta_{\text{New Equity}}$ = 3.035

iii. Yes, it justifies the increase as it leads to increase in the Value of Equity due to increase in Beta.

Question 35:

i. Determination of Beta of Car AC and Window AC

$$\frac{\sigma_{sm}\sigma_s}{\sigma_m}$$

Car AC

$$\frac{0.6 \times 0.3}{0.2} = 0.90$$

Window AC

$$\frac{0.7 \times 0.35}{0.2} = 1.225$$

Beta of Split AC/ Window AC is

$$\frac{0.85 \times 0.50}{0.2} = 2.125$$

The Beta of Split AC alone is

2.125 = $0.50\beta_S + 0.50\beta_c$

= $0.50\beta_S$ + 0.50 x 1.225

β_S = 3.025

ABC Ltd.'s Beta shall be:

0.6 x 0.9 + 0.25 x 1.225 + 0.15 x 3.025 = 1.30

ii. Cost of Equity of ABC Ltd.

K_e = 4% + 1.30(10% - 4%) = 11.80%

iii. Calculation of Debt Beta

$$\frac{5\% - 4\%}{10\% - 4\%} = 0.167$$

Accordingly, Beta of Equity shall be

1.30 = 0.50 x 0.167 + 0.50 x β_e

= 2.433

Question 36:

a. Let the weight of stocks of Economy A is expressed as w, then

(1- w)×10.0 + w ×15.0 = 10.5
i.e. w = 0.1 or 10%.

b. Variance of portfolio shall be:

$(0.9)^2 (0.16)^2 + (0.1)^2 (0.30)^2 + 2(0.9) (0.1) (0.16) (0.30) (0.30) = 0.02423$
Standard deviation is $(0.02423)^{½} = 0.15565$ or 15.6%.

c. The Sharpe ratio will improve by approximately 0.04, as shown below:

$$\text{Sharpe Ratio} = \frac{\text{Expected Return - Risk Free Rate of Return}}{\text{Standard Deviation}}$$

Investment only in developed countries: (10 – 3)/16 = 0.437
With inclusion of stocks of Economy A: (10.5 – 3)/ 15.6 = 0.481

Question 39:

i. Sensitivity of each stock with market is given by its beta.
Standard deviation of market Index = 15%
Variance of market Index = 0.0225
Beta of stocks = σ_i r/ σ_m
A = 20 × 0.60/15 = 0.80
B = 18 × 0.95/15 = 1.14
C = 12 × 0.75/15 = 0.60

ii. Covariance between any 2 stocks = $\beta 1 \beta_2 \sigma^2_m$

Covariance matrix

Stock/Beta	0.80	1.14	0.60
A	400.000	205.200	108.000
B	205.200	324.000	153.900
C	108.000	153.900	144.000

iii. Total risk of the equally weighted portfolio (Variance)

$= 400(1/3)^2 + 324(1/3)^2 + 144(1/3)^2 + 2 (205.20)(1/3)^2 + 2(108.0)(1/3)^2 + 2(153.900) (1/3)^2 =$ 200.244

iv. β of equally weighted portfolio =

$\beta_p = \sum \beta_i/N = \frac{0.80+1.14+0.60}{3}$

= 0.8467

v. Systematic Risk $\beta_P^2\sigma_m^2$ = $(0.8467)^2\ (15)^2$ =161.303

Unsystematic Risk = Total Risk – Systematic Risk

= 200.244 – 161.303 = 38.941

Question 40:

i.

Security	Expected Return	Beta (β)	Required Return =0.08 + 0.04β	Under Valued Security
O	0.32	1.70	0.148	UVS
P	0.30	1.40	0.136	UVS
Q	0.25	1.10	0.124	UVS
R	0.22	0.95	0.118	UVS
S	0.20	1.05	0.122	UVS
T	0.14	0.70	0.108	UVS
U				

UVS = Under Valued Security

All the securities listed above are undervalued because their expected returns plot above the SML

ii. a. Expected return on the portfolio

$$= \frac{1}{6}\ (0.32 + 0.30 + 0.25 + 0.22 + 0.20 + 0.14) = 0.2383$$

b. Expected return on the portfolio

$R_P = XR_M - (X-1)\ R_P$

= (1.4) (0.2383) – (0.4) (0.12) = 0.33362 – 0.048 = 0.28562

Question 47:

i. Mr. X's position in the two securities are +1.50 in security A and -0.5 in security B. Hence the portfolio sensitivities to the two factors:-

b prop. 1 =1.50 x 0.80 + (-0.50 x 1.50) = 0.45

b prop. 2 = 1.50 x 0.60 + (-0.50 x 1.20) = 0.30

ii. Mr. X's current position:-

Security A ₹ 3,00,000 / ₹ 1,00,000 = 3

Security B -₹ 1,00,000 / ₹ 1,00,000 = -1

Risk free asset -₹ 100000 / ₹ 100000 = -1

b prop. 1 = 3.0 x 0.80 + (-1 x 1.50) + (- 1 x 0) = 0.90

b prop. 2 = 3.0 x 0.60 + (-1 x 1.20) + (-1 x 0) = 0.60

iii. Expected Return = Risk Free Rate of Return + Risk Premium

Let λ_1 and λ_2 are the Value Factor 1 and Factor 2 respectively.

Accordingly

$15 = 10 + 0.80\ \lambda_1 + 0.60\ \lambda_2$
$20 = 10 + 1.50\ \lambda_1 + 1.20\ \lambda_2$

On solving equation, the value of $\lambda_1 = 0$, and risk premium of factor 2 for Securities A & B shall be as follows:

Using Security A's Return

Total Return = 15% = 10% + 0.60 λ_2
Risk Premium (λ_2) = 5%/ 0.60 = 8.33%

Alternatively using Security B's Return

Total Return = 20% = 10 + 1.20 λ_2
Risk Premium = 10%/ 1.20 = 8.33%

Question 49:

Return of the stock under APT

Factor	Actual value in %	Expected value in %	Difference	Beta	Diff. x Beta
GNP	7.70	7.70	0.00	1.20	0.00
Inflation	7.00	5.50	1.50	1.75	2.63
Interest rate	9.00	7.75	1.25	1.30	1.63
Stock index	12.00	10.00	2.00	1.70	3.40
Ind. Production	7.50	7.00	0.50	1.00	0.50
					8.16
Risk free rate in %					9.25
Return as per APT					17.41

Question 50:

Security	$\frac{R_i - R_f}{\beta_i}$	$\frac{(R_i - R_f)\times\beta_i}{\sigma^2_{ei}}$	$\sum_{i=1}^{N}\frac{(R_i - R_f)\times\beta_i}{\sigma^2_{ei}}$	$\frac{\beta_i^2}{\sigma^2_{ei}}$	$\sum_{i=1}^{N}\frac{\beta_i^2}{\sigma^2_{ei}}$	C_i
1	14	0.7	0.7	0.05	0.05	4.67
2	12	0.9	1.6	0.075	0.125	7.11
3	12	0.3	1.9	0.025	0.15	7.60
4	10	1.0	2.9	0.1	0.25	8.29
5	8	0.4	3.3	0.05	0.3	8.25
6	8	0.04	3.34	0.005	0.305	8.25
7	6	0.45	3.79	0.075	0.38	7.90

'C_i' calculations are given below:

For Security 1

$$C_1 = \frac{10 \times .7}{1 + 10(.05)} = 4.67$$

Here 0.7 is got from column 4 and 0.05 from column 6. Since the preliminary calculations are over, it is easy to calculate the C_i.

$$C_2 = \frac{10 \text{ x } 1.6}{1 + 10\ (.125)} = 7.11$$

$$C_3 = \frac{10 \text{ x } 1.9}{1 + 10\ (0.15)} = 7.6$$

$$C_4 = \frac{10 \times 2.9}{1 + 10(0.25)} = 8.29$$

$$C_5 = \frac{10 \times 3.3}{1 + 10\ (0.3)} = 8.25$$

$$C_6 = \frac{10 \times 3.34}{1 + 10\ (0.305)} = 8.25$$

$$C_7 = \frac{10 \times 3.79}{1 + 10(0.38)} = 7.90$$

The highest C_i value is taken as the cut-off point i.e. C*. The stocks ranked above C* have high excess returns to beta than the cut-off C and all the stocks ranked below C* have low excess returns to beta. Here, the cut-off point is 8.29. Hence, the first four securities i.e. 1 – 4 are selected and remaining 3 are rejected.

Now we shall compute how much to be invested in each security by calculating Z_i for these four securities as follows:

$$Z_i = \frac{B_i}{\sigma_i^2}\left(\frac{R_i - R_o}{{}_1B_i} - C*\right)$$

Thus,

$$Z_1 = \frac{1.00}{20}\left(\frac{14}{1.0} - 8.29\right) = 0.05(5.71) = 0.2855$$

$$Z_2 = \frac{1.5}{30}\left(\frac{18}{1.5} - 8.29\right) = 0.05(3.71) = 0.1855$$

$$Z_3 = \frac{0.5}{10}\left(\frac{6}{0.5} - 8.29\right) = 0.05(3.71) = 0.1855$$

$$Z_4 = \frac{2}{40}\left(\frac{20}{2} - 8.29\right) = 0.05(1.71) = 0.0855$$

The proportion of investment in each stock will be computed as follows:

$$X_i = \frac{Z_i}{\sum_{i=1}^{n} Z_j}$$

Thus $\sum_{i=1}^{n} Z_j$ = 0.2855 + 0.1855 + 0.1855 + 0.0855 = 0.742

Accordingly, proportion of investments in

Security 1 $= \frac{0.2855}{0.742} = 0.3848$ i.e. 38.48%

Security 2 $= \frac{0.1855}{0.742} = 0.25$ i.e. 25%

Security 3 $= \frac{0.1855}{0.742} = 0.25$ i.e. 25%

Security 4 $= \frac{0.0855}{0.742} = 0.1152$ i.e. 11.52%

Thus investment as per following proportion will be the optimal portfolio.
Security 1 → 38.48%
Security 2 → 25%
Security 3 → 25%
Security 4 → 11.52%

Question 52:

Date	Closing Sensex	Sign of Price Charge
1.10.07	2800	
3.10.07	2780	-
4.10.07	2795	+
5.10.07	2830	+
8.10.07	2760	-
9.10.07	2790	+
10.10.07	2880	+
11.10.07	2960	+
12.10.07	2990	+
15.10.07	3200	+
16.10.07	3300	+
17.10.07	3450	+

19.10.07	3360	-
22.10.07	3290	-
23.10.07	3360	+
24.10.07	3340	-
25.10.07	3290	-
29.10.07	3240	-
30.10.07	3140	-
31.10.07	3260	+

Total of sign of price changes (r) = 8

No of Positive changes = n_1 = 11

No. of Negative changes = n_2 = 8

$$\mu_r = \frac{2n_1n_2}{n_1+n_2}+1$$

$$\mu = \frac{2\times11\times8}{11+8}+1 = 176/19+1 = 10.26$$

$$\hat{\sigma}_r = \sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}}$$

$$\hat{\sigma}_r = \sqrt{\frac{(2\times11\times8)(2\times11\times8-11-8)}{(11+8)^2(11+8-1)}} = \sqrt{\frac{176\times157}{(19)^2(18)}} = \sqrt{4.252} = 2.06$$

Since too few runs in the case would indicate that the movement of prices is not random. We employ a two- tailed test the randomness of prices.

Test at 5% level of significance at 18 degrees of freedom using t- table

The lower limit

$$= \mu - t\times\hat{\sigma}_r = 10.26 - 2.101\times2.06 = 5.932$$

Upper limit

$$= \mu + t\times\hat{\sigma}_r = 10.26 + 2.101\times2.06 = 14.588$$

At 10% level of significance at 18 degrees of freedom

Lower limit

= 10.26 – 1.734 × 2.06 = 6.688

Upper limit

= 10.26 + 1.734 × 2.06 = 13.832

As seen r lies between these limits. Hence, the market exhibits weak form of efficiency.

*For a sample of size n, the t distribution will have n-1 degrees of freedom.

Question 53:

Months (I)	Actual Return on Reddy's stock (%) (II)	Return on sensex (%) (III)	Expected return on Reddy's stock using characteristic line (IV)	Above normal return % (V) = (II) – (IV)
January 2001	–	–	–	
February, 2001	2.53	–1.84	2.98	–0.45
March, 2001	–6.42	–15.13	–3.80	–2.62
April, 2001	–10.08	–2.36	2.72	–12.80
May, 2001	25.39	3.20	5.55	19.84
June, 2001	14.50	–4.82	1.46	13.04
July, 2001	4.85	–3.69	2.04	2.81
August, 2001	4.92	–2.53	2.63	2.29
September, 2001	0.79	–13.35	–2.89	3.68
October, 2001	–41.46	6.32	7.14	–48.60
			Σ Above normal return =	–22.81%

From the above computation we observe that sum of abnormal return is not close to zero. Therefore, we conclude that market is not efficient in semi-strong form.

Topic 21 EVENT STUDIES

Question 54:

First we should find out abnormal return by deducting the actual return from the expected return

Star Software Limited

Period	Actual return (rSt)	Market return (rmt)	Expected return (%) (1.25 + 0.92 rmt)	Abnormal return
3	11.21	10.25	10.68	0.53
2	11.42	10.75	11.14	0.28
1	12.02	10.90	11.28	0.74
0	11.98	10.80	11.19	0.79
1	12.92	11.25	11.60	1.32
2	12.23	10.92	11.30	0.93
3	12.75	11.15	11.51	1.24

Kanishka Airways

Period	Actual return (r_{Kt})	Market return (r_{mt})	Expected return $1.39 + 1.03\ (r_{mt})$	Abnormal return
3	11.78	10.25	11.95	– 0.17
2	12.49	10.75	12.46	0.03
1	13.02	10.90	12.62	0.40
0	12.29	10.80	12.51	– 0.22
1	13.45	11.25	12.98	0.47
2	13.02	10.92	12.64	0.38
3	13.21	11.15	12.87	0.34

Indian Auto

Period	Actual return	Market return	Expected return	Abnormal return
	($r_{I,t}$)	(l_{mt})	($1.78 + 1.07\ r_{mt}$)	
3	12.25	10.25	12.75	– 0.50
2	13.25	10.75	13.28	– 0.03
1	13.39	10.90	13.44	– 0.05
0	13.10	10.80	13.34	– 0.24
1	14.25	11.25	13.82	0.43
2	14.01	10.92	13.46	0.55
3	14.19	11.15	13.71	0.48

We will now estimate the average abnormal return to each of the months before and after the dividend was announced

Third month before the announcement of dividend

$$AAR_{(-3)} = \frac{1}{3}(0.53 - 0.17 - 0.50) = -0.0467$$

Second month before the announcement of dividend

$$AAR_{(-2)} = \frac{1}{3}(0.28 + 0.03 - 0.03) = 0.093.$$

First month before the announcement of dividend

$$AAR_{(-1)} = \frac{1}{3}(0.74 + 0.40 - 0.05) = 0.363$$

Month during which the dividend was announced

$$AAR_{(0)} = \frac{1}{3}(0.79 - 0.22 - 0.24) = 0.11$$

First month after the announcement of dividend

$$AAR_{(1)} = \frac{1}{3}(1.32 + 0.47 + 0.43) = 0.74.$$

Second month after the announcement of dividend

$$AAR_{(2)} = \frac{1}{3}(0.93 + 0.38 + 0.55) = 0.62$$

Third month after the announcement of dividend

$$AAR_{(3)} = \frac{1}{3}(1.24 + 0.34 + 0.48) = 0.686.$$

Now we will compute the cumulative Average Abnormal returns for the period of three months before and after the announcement of dividend.

$$CAAR = (-0.0467 + 0.093 + 0.363 + 0.11 + 0.74 + 0.62 + 0.686) = 2.5653\%.$$

As the value of CAAR is not close to zero, we conclude that market is not efficient in the semi-strong form.

Question 55:

Maximum decline in one month $= \frac{5326 - 4793.40}{5326} \times 100 = 10\%$

i. Immediately to start with

Investment in equity = Multiplier x (Portfolio value – Floor value)
= 2 (3,00,000 – 2,70,000) = ₹ 60,000

Indira may invest ₹ 60,000 in equity and balance in risk free securities.

ii. After 10 days

Value of equity = 60,000 x 5122.96/5326	₹ 57,713
Value of risk free investment	₹ 2,40,000
Total value of portfolio	**₹ 2,97,713**
Investment in equity = Multiplier x (Portfolio value – Floor value) = 2 (2,97,713 – 2,70,000)	₹ 55,426
Revised Portfolio	
Equity	₹ 55,426
Risk free Securities = ₹ 2,97,713 – ₹ 55,426	₹ 2,42,287

iii. After another 10 days

Value of equity = 55,426 x 5539.04/5122.96	₹ 59,928
Value of risk free investment	₹ 2,42,287
Total value of portfolio	**₹ 3,02,215**
Investment in equity = Multiplier x (Portfolio value – Floor value) = 2 (3,02,215 – 2,70,000)	₹ 64,430
Revised Portfolio	
Equity	₹ 64,430
Risk Free Securities = ₹ 3,02,215 – ₹ 64,430	₹ 2,37,785

The investor should off-load ₹ 4502 of risk free securities and divert to Equity

Question 56:

Constant Ratio Plan:

Stock Portfolio NAV (₹)	Value of Conservative Portfolio (₹)	Value of aggressive Portfolio (₹)	Total value of Constant Ratio Plan (₹)	Revaluation Action	Total No. of units in aggressive portfolio
40.00	10,00,000	10,00,000	20,00,000	-	25000
25.00	10,00,000	6,25,000	16,25,000	-	25000
	8,12,500	8,12,500	16,25,000	Buy 7500 units	32500
36.00	8,12,500	11,70,000	19,82,500	-	32500
	9,91,250	9,91,250	19,82,500	Sell 4965.28 units	27534.72
32.00	9,91,250	8,81,111.04	18,72,361.04	-	27534.72
38.00	9,91,250	10,46,319.36	20,37,569.36	-	27534.72
	10,18,784.68	10,18,784.68	20,37,569.36	Sell 724.60 units	26810.12
37.00	10,18,784.68	9,91,974.44	20,10,759.12	-	26810.12
42.00	10,18,784.68	11,26,025.04	21,44,809.72	-	26810.12
43.00	10,18,784.68	11,52,835.16	21,71,619.84	-	26810.12

Hence, the ending value of the mechanical strategy is ₹ 21,71,619.84 and buy & hold strategy is ₹ 21,50,000.

Question 57:

i. **Investment committed to each security would be:**

	A (₹)	B (₹)	C (₹)	Total (₹)
Portfolio X	1,500	2,000	1,500	5,000
Portfolio Y	600	1,500	900	3,000
Combined Portfolio	2,100	3,500	2,400	8,000
∴Stock weights	0.26	0.44	0.30	

ii. **The equation of critical line takes the following form:**

WB = a + bWA

Substituting the values of WA & WB from portfolio X and Y in above equation, we get

0.40 = a + 0.30b, and
0.50 = a + 0.20b
Solving above equation we obtain the slope and intercept, a = 0.70 and b= -1 and thus, the critical line is

WB = 0.70 – WA

If half of the funds is invested in security A then,

WB = 0.70 – 0.50 = 0.20

Since WA + WB + WC = 1

WC = 1 - 0.50 – 0.20 = 0.30

∴ Allocation of funds to

Security B	= 0.20 x 8,000 = ₹ 1,600,
&	
Security C	= 0.30 x 8,000 = ₹ 2,400

Question 59:

i. Variance of Returns

$$Cor_{i,j} = \frac{Cov(i,j)}{\sigma_i \sigma_j}$$

Accordingly, for MFX

$$1 = \frac{Cov(X,X)}{\sigma_X \sigma_X}$$

$$\sigma_x^2 = 4.800$$

Accordingly, for MFY

$$1 = \frac{Cov(Y,Y)}{\sigma_Y \sigma_Y}$$

$$\sigma_Y^2 = 4.250$$

Accordingly, for Market Return

$$1 = \frac{Cov(M,M)}{\sigma_M \sigma_M}$$

$$\sigma_M^2 = 3.100$$

ii. Portfolio return, beta, variance and standard deviation

$$\text{Weight of MFX in portfolio } = \frac{1,20,000}{2,00,000} = 0.60$$

$$\text{Weight of MFY in portfolio } = \frac{80,000}{2,00,000} = 0.40$$

Accordingly Portfolio Return

0.60 × 15% + 0.40 × 14% = 14.60%

$$\text{Beta of each Fund } \beta = \frac{Cov(Fund,Market)}{Variance\ of\ Market}$$

$$\beta_X = \frac{3.370}{3.100} = 1.087$$

$$\beta_Y = \frac{2.800}{3.100} = 0.903$$

Portfolio Beta

0.60 x 1.087 + 0.40 x 0.903 = 1.013

Portfolio Variance

$$\sigma^2_{XY} = W^2_X\sigma^2_X + W^2_Y\sigma^2_Y + 2W_XW_YCov_{X,Y}$$

= $(0.60)^2$ (4.800) + $(0.40)^2$ (4.250) + 2(0.60) (0.40) (4.300)
= 4.472

Or Portfolio Standard Deviation

$$\sigma_{XY} = \sqrt{4.472} = 2.115$$

iii. Expected Return, Systematic and Unsystematic Risk of Portfolio

Portfolio Return = 10% + 1.0134(12% - 10%) = 12.03%
MF X Return = 10% + 1.087(12% - 10%) = 12.17%
MF Y Return = 10% + 0.903(12% - 10%) = 28.06%
Systematic Risk = β^2 σ^2

Accordingly,

Systematic Risk of MFX = $(1.087)^2$ x 3.10 = 3.663
Systematic Risk of MFY = $(0.903)^2$ x 3.10 = 2.528
Systematic Risk of Portfolio = $(1.013)^2$ x 3.10 = 3.181
Unsystematic Risk = Total Risk – Systematic Risk

Accordingly,

Unsystematic Risk of MFX = 4.80 – 3.663 = 1.137
UnSystematic Risk of MFY = 4.250 – 2.528 = 1.722
UnSystematic Risk of Portfolio = 4.472 – 3.181 = 1.291

iv. Sharpe and Treynor Ratios and Alpha

Sharpe Ratio

$$MFX = \frac{15\% - 10\%}{\sqrt{4.800}} = 2.282$$

$$MFY = \frac{14\% - 10\%}{\sqrt{4.250}} = 1.94$$

$$Portfolio = \frac{14.6\% - 10\%}{2.115} = 2.175$$

Treynor Ratio

$$MFX = \frac{15\% - 10\%}{1.087} = 4.60$$

$$MFY = \frac{14\% - 10\%}{0.903} = 4.43$$

$$Portfolio = \frac{14.6\% - 10\%}{1.0134} = 4.54$$

Alpha

MFX = 15% - 12.17% = 2.83%

MFY = 14% - 11.81% = 2.19%

Portfolio = 14.6% - 12.03% = 2.57%

Question 67:

First of we shall calculate expected return from share of Company X

i. Average annual capital gain (%)

Let g = average annual capital gain, then:

₹ 203.51(1+g)^{¼} = ₹139

Then g = (203.51/139)^{¼} -1 = 0.10 i.e. 10%

ii. Average annual dividend yield (%)

Year	Dividend/Share Price	Dividend Yield
2010	₹7.00/₹139	0.050
2011	₹8.50/ ₹147	0.058
2012	₹9.00/ ₹163	0.055
2013	₹9.50/ ₹179	0.053
2014 (Current Year)	₹10.00/ ₹203.51	0.049
		0.265

Average Yield = 0.265/5= 0.053 i.e. 5.3%

Thus with this data expected return of share of Company X can be given as follows:

$E(r_X)$ = Average Annual Capital Gain + Average Annual Dividend

= 10% + 5.3% = 15.3%

Then we shall calculate expected return from market index as follows:

i. Average annual capital gain (%)

1300 (1+g)^{¼} = 1768

Then g = (1768/1300)^{¼} -1 = 0.08 i.e. 8%

ii. Average annual dividend yield (%)

3% + 5% + 5.5% + 4.75% + 5.5% = 23.75%/5 = 4.75%

Thus expected return on Market Index $E(r_M)$ = 8% + 4.75% = 12.75%

Average annual risk-free rate of return (Treasury Bond Return)

7% + 9% + 8% + 8% + 8% = 40%/5 = 8%

Now with the above information we compute Beta (β) of share company X using CAPM as follows:

$E(r_X) = r_f + \beta[E(r_M) - r_f]$

15.3% = 8% + β[12.75% - 8%]

β = 1.54

Question 71:

If it is assumed 50% investment in each of the two securities then Return and Risk of Portfolio shall be computed as follows:

Year	Return of L	Deviation $\left(R_L - \overline{R_L}\right)$	Deviation $\left(R_L - \overline{R_L}\right)^2$	Return of K	Deviation $\left(R_K - \overline{R_K}\right)$	Deviation $\left(R_K - \overline{R_K}\right)^2$	Product of deviations
2012	10	1	1	11	3	9	3
2013	04	-5	25	-6	-14	196	70
2014	05	-4	16	13	5	25	-20
2015	11	2	4	8	0	0	0
2016	15	6	36	14	6	36	36
	Σ = 45		Σ= 82	Σ=40		Σ=266	
$\overline{R_L} = \frac{45}{5} = 9$			$\overline{R_K} = \frac{40}{5} = 8$				89

Covariance = $\frac{\sum_{i=1}^{N}\left[R_1 - \overline{R_1}\right]\left[R_2 - \overline{R_2}\right]}{N} = 89/5 = 17.8$

Return and Standard Deviation of Security L

$$R_L = \frac{45}{5} = 9$$

$$\sigma_L = \sqrt{\frac{\left(R_{^L} - \overline{R_L}\right)^2}{N}}$$

$$\sigma_L = \sqrt{\frac{82}{5}} = 4.05$$

Standard Deviation of Security K

$$\sigma_K = \sqrt{\frac{\left(R_K - \overline{R_K}\right)^2}{N}}$$

$$\sigma_K = \sqrt{\frac{266}{5}} = 7.29$$

Portfolio Return

R_P = 0.50 x 9 +0.50 x 8 = 8.50%

Portfolio Standard Deviation

$\sigma_{LK} = (0.50^2 X 4.05^2 + 0.50^2 X 7.29^2 + 2X 0.5 X 0.5 X 17.8)^{½} = 5.12$